

$$U_{p2} = U_{p3}. \quad (18)$$

From Eq. (4)

$$P_1 - P_0 = \rho_0 (U_{s1} - U_{p0}) (U_{p1} - U_{p0}),$$

$$P_2 - P_0 = \rho'_0 (U_{s2} - U_{p0}) (U_{p2} - U_{p0}),$$

$$P_3 - P_1 = \rho_1 (U_{s3} - U_{p1}) (U_{p3} - U_{p1}).$$

The material ahead of the shock waves U_{s1} and U_{s2} is assumed at rest so that $P_0 = U_{p0} = 0$. Using the continuity conditions and the first and third of the above equations,

$$P_2 = \rho_0 U_{s1} U_{p1} + \rho_1 (U_{s3} - U_{p1}) (U_{p2} - U_{p1}).$$

Substituting for P_2 and simplifying

$$\frac{U_{p2}}{U_{p1}} = \frac{\rho_0 U_{s1} - \rho_1 (U_{s3} - U_{p1})}{\rho'_0 U_{s2} - \rho_1 (U_{s3} - U_{p1})} \quad (19)$$

and

$$\frac{P_2}{P_1} = \frac{\rho'_0 U_{s2} [\rho_0 U_{s1} - \rho_1 (U_{s3} - U_{p1})]}{\rho_0 U_{s3} [\rho'_0 U_{s2} - \rho_1 (U_{s3} - U_{p1})]}. \quad (20)$$

The shock waves moving to the right, such as U_{s1} and U_{s2} are defined as traveling in the positive direction and those to the left, such as U_{s3} are moving in the negative direction. In an experiment, the impedances $\rho_0 U_{s1}$ and $\rho'_0 U_{s2}$ for the media are readily determined from the measured shock velocities. However, the quantity $\rho_1 U_{s3}$ is very troublesome due to the difficulty in measuring U_{s3} by the technique used here. This problem can be overcome if an extension of the acoustic approximation¹⁹ is used to write

$$\rho_0 U_{s1} = -\rho_1 (U_{s3} - U_{p1}). \quad (21)$$

Equations (19) and (20) can then be simplified to read